

Object Segmentation Based on Location Information for Level Set Method

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ABSTRACT

In this paper, a novel level set approach is presented to segment only indicated objects from natural color images. To solve this challenging problem, users first initialize a contour inside the concerned object. The proposed level set model is then applied to achieve the object containing this initial region regardless of its shape and size. In our novel framework, the energy functional is a combination of two energy terms based on Bhattacharya flow and graph partitioning active contour. So that movement of the contour can be controlled toward object boundaries by correlative relationships between the interested region and surroundings. The experimental results obtained from our natural image collection show that the suggested method yields more accurate and better performance when comparing to other segmentation algorithms.

Categories and Subject Descriptors

I.4.6 [Image Processing and Computer Vision]: Segmentation – edge and feature detection, region growing, partitioning.

General Terms

Algorithms.

Keywords

Object segmentation, location information, Bhattacharyya flow, graph partitioning, level set.

1. INTRODUCTION

Image segmentation plays an important role in computer vision for providing the most valuable information of images. A significant growth in the number of segmentation algorithms has taken place. But it is a hard task due to real-world variations in color distribution, object category, position, size, etc.. Recently, variational methods have been extensively studied for detecting all salient objects because of their flexible modeling and easy numerical implementation. However, if users would like to segment only one object of interest at a specified location in the image, segmentation using level set approaches still exists its own

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difficulties as well as limitations illustrated in Fig. 1.

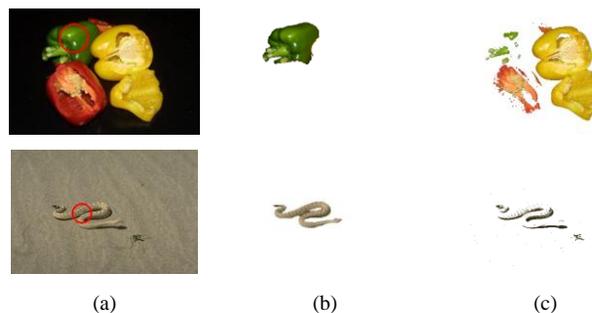


Fig. 1. Limitations of level set methods for segmenting only object of interest, (a): color images with user-initialized contour, (b): desired results, (c): wrong segmentation results

Compared to other works, our contribution in this paper is to modify the energy functional described in the proposed level set method. Based on its changes, a distance relationship measured between two any pixels is added to constrain our model for segmenting only one object of interest in the location indicated manually by users. Moreover, the object can be extracted if its colors or textures are slightly different. In this study, the term “one object” is to indicate a single object or many objects that are stuck together and have similar properties as shown in Fig. 2. The second term is “location information”. It is not enough to segment only the object of interest using colors. Since there can be objects which have the same color but are in different location, this information of user-initialized contour is used to avoid this wrong object segmentation.

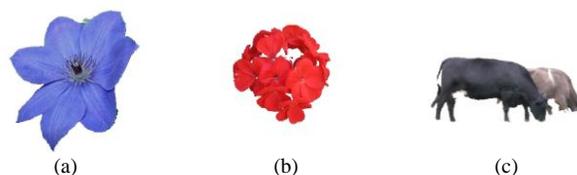


Fig. 2. Illustrations of the definition of the term “one object”, (a): “One object” is a single flower, (b): “One object” is a bunch of red small flowers, (c): This case is not considered as “one object”

The remainder of this paper is organized as follows. Section 2 reviews some related works. Our method and experimental results are discussed in Sections 3 and 4, respectively. Finally, Section 5 gives conclusions.

2. RELATED WORK

In general, many works have been reported about image segmentation extensively. Commonly a new segmentation algorithm usually starts as gray-level segmentation and is later developed to handle color and texture images. Traversing the huge amount of existing techniques, they can be categorized into four types of segmentation: thresholding [1, 2], boundary-based [3, 4], region-based [5, 6], and hybrid algorithms [7, 8].

In particular, for better segmenting dynamic-shape objects in medical images, the active contour model (ACM) was first introduced by Kass, Witkins, and Terzopoulos [9] in 1987 as an interactive segmentation model for 2D images. It is also called the snakes or deformable models. It started with a contour drawn by the user and iteratively deformed under an energy minimization framework to get the final boundary contour. Subsequently, Osher and Sethian [10] first proposed the level set method in 1988. Its basic idea is to embed a 2D contour in a surface in 3D space. So the solution has become to control the curve evolution rather than only track object boundaries.

According to the nature of color images, it is considerable to inherit strong points of the existing ACMs with more information cues, to lead to effective segmentation algorithms [11-14]. For the growth of this research, combinations of ACMs and other segmentation methods gave new strategies to solve this problem. Variational cost functions can be defined by integrating with stochastic representations [15, 16], graph partitioning methods [17, 18], generic algorithm [19], etc.. Among these models, graph partitioning active contour (GPAC) proposed by Sumengen and Manjunath in 2006 [20] has the closest framework to the C-V model. Its concept is to reformulate the problem in a continuous domain based on the minimum-cut formulation and solve it using the level set framework, rather than graph-cuts. Hence, an improvement of a GPAC model could be considered for our particular problem, to suit our requirements as well as increase its performance related to convergence speed and accuracy.

3. LOCATION BASED LEVEL SET METHOD FOR OBJECT SEGMENTATION

3.1 System Overview

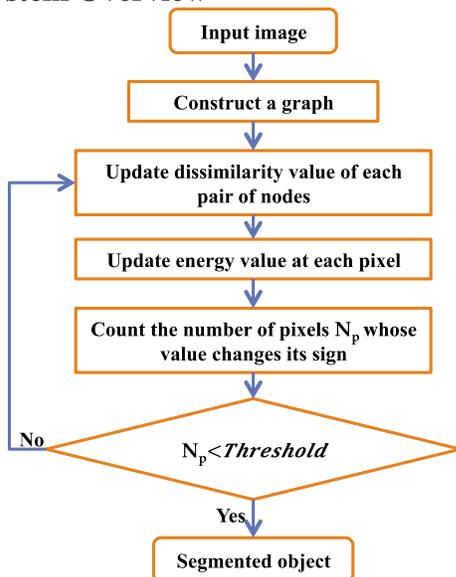


Fig. 3. Proposed method flowchart

A flowchart of the proposed method is depicted in Fig. 3. We first construct a graph whose nodes are pixels of the input image, and define edge weight between two any nodes by a dissimilarity value. A process of finding a bounding contour of the concerned object is then repeated. In each iteration, the energy functional of the proposed level set method is updated. A number of pixels whose energy value changes its sign is then counted and compared with a threshold. If it is larger than the threshold, the process will be continued by updating dissimilarity value in the graph. Otherwise, the object of interest is segmented.

3.2 Graph Partitioning Active Contour

Sumengen and Manjunath introduced a new curve evolution framework called Graph Partitioning Active Contours (GPAC) in [20]. This framework is motivated by the relationship between active contours and graph theoretic methods. The aim of this combination is to minimize pairwise similarities between points on the curve C or maximize dissimilarities of across-region cuts. In continuous domain, a variational cost function of curve evolution based on minimum cut criterion can be formulated as

$$E_{cr}(C) = \iint_{R_i(C)} \iint_{R_o(C)} w(u,v) du dv \quad (1)$$

where $R_i(C)$ and $R_o(C)$ are the interior and exterior regions of C , u and v are points such that $u \in R_i$, $v \in R_o$, and $\omega(u,v)$ is a similarity measure between points u and v . Minimization of $E_{cr}(C)$ with respect to the curve C results in partitioning of the image into the two most dissimilar regions. Using the steepest descent method where a curve is instantiated and evolved toward the expected minimum, the problem will be solved. L_p -norms, L_1 and L_2 are usually applied to compute color features. Without any loss of generality, $\omega(u,v)$ can be dissimilarity metric and GPAC can use more complex measures that combine spatial distance of pixels and domain knowledge.

Applying to object segmentation problems, the GPAC has many advantages [20]. One of them is that some geometric properties or constraints can be introduced into the curve evolution equation without changing or resolving the energy minimization problem due to the use of geometric ACM. Secondly, the theory developed in this variational framework can be easily adapted and applied to other cost functions that are based on pairwise (dis)similarities. Furthermore, GPAC is a region-based model for segmentation; but it can offer a flexible framework where edge information can be integrated to help extract more precise boundaries. Since then, a lot of modifications can be flexibly considered for GPAC to increase its performance the most [21, 22].

3.3 The Bhattacharyya Flow

In [23] the Bhattacharyya distance formally gives a similarity measure between two probability distributions, defined as $D = -\log B$, where B is the Bhattacharyya coefficient

$$B = \int_Z \sqrt{P_{in}(z)P_{out}(z)} dz \quad (2)$$

In image processing field, $z \in Z$ is a photometric variable such as intensity, a color vector, or a texture vector, and lives in Z , the space of the photometric variable. P_{in} and P_{out} are probability distributions defined on the variable z for the inside and outside

regions, respectively. In Eq. 2, maximizing the Bhattacharyya distance D is equivalent to minimizing B . Thus, the final contour in ACMs is obtained when the overlap between the intensity distributions of object and background (inside and outside the contour) is minimized.

Let $x \in \mathcal{R}^2$ specify the coordinates in the image plane, and $I: \Omega \subset \mathcal{R}^2 \rightarrow Z$ be a mapping from the image plane to the space of the photometric variable. For the case of curve evolution, the probability distributions P_{in} and P_{out} are assumed to be defined by the density of the region inside the curve C . Thus, in terms of the level set function ϕ , we get

$$P_{in}(z) = \frac{\int_{\Omega} K(z - I(x))H(-\phi(x))dx}{\int_{\Omega} H(-\phi(x))dx} \quad (3)$$

$$P_{out}(z) = \frac{\int_{\Omega} K(z - I(x))H(\phi(x))dx}{\int_{\Omega} H(\phi(x))dx} \quad (4)$$

where H is the Heaviside step function and Ω is the whole image domain.

3.4 Location Based Level Set Method for Object Segmentation

It is not a trivial task to not only identify and extract objects of interest out of the original image but also remove similar characteristic surrounding regions. Generally, our proposed algorithm is a combination of ideas of two approaches, the Bhattacharyya flow and the GPAC, inspired by their own advantages. The level set method is commonly defined by a particular energy functional describing features of the object we want to segment. So that, the energy functional here includes two primary elements: a local energy based on GPAC and a global energy based on the Bhattacharyya flow.

In this work, for the local energy term, our basic variational cost oppositely relies on within region dissimilarity and the evolution of the front C is done implicitly using the level set framework. Therefore, the local energy term takes into account the dissimilarity within each region. For the global energy term, the Bhattacharyya flow helps measure the overlap between the intensity distributions of the background and foreground regions. Consequently, not only the difference of the intra-regions is minimized but the distance between the two regions is maximized as well. Essentially, our fitting energy functional is developed as the following

$$E(C) = \beta E_{local}(C) + (1 - \beta) E_{global}(C) \quad (5)$$

where β is the balance coefficient $0 < \beta < 1$ to control the effect of the local and global terms. In Eq. 5, the local energy term is chosen as Bertelli et al. reformulated in [21].

$$E_{local}(C) = E_{WR}(C) = \iint_{R_i(C) R_o(C)} w(u, v) du dv + \iint_{R_o(C) R_i(C)} w(u, v) du dv \quad (6)$$

The global energy term here is chosen as the Bhattacharyya distance. Because this calculation is applied in color images, the Bhattacharyya coefficient is rewritten as the summation of B values computed from both three color channels

$$E_{global}(C) = E_{Bha}(C) = -\sum_{i=1}^3 B^i(C) \quad (7)$$

To be suitable with the nature of color images, the dissimilarity measure in Eq. 6 uses L_2 distance on color and location features as below

$$\omega(u, v) = \frac{\|I(u) - I(v)\|}{\sqrt{3}} + \frac{1}{l_{diag}^2} \times \begin{cases} 0 & u, v \in RIC \\ \text{dist}(u, RIC) & u \notin RIC, v \in RIC \\ \text{dist}(v, RIC) & u \in RIC, v \notin RIC \\ \text{dist}(u, RIC)\text{dist}(v, RIC) & u \notin RIC, v \notin RIC \end{cases} \quad (8)$$

where RIC is the abbreviation of ‘‘region inside curve’’, l_{diag}^2 is the square of length of the diagonal of the image, and $\text{dist}(u, RIC) = \min \text{dist}(u, x)$ for all points $x \in \partial C$.

While the above presented cost function demonstrates a great potential, the trouble still exists that the contributions of within region dissimilarity of different regions (i.e. inside and outside the curve) in the level set formulation are the same. It can compound the slowness of contours evolution (or iterative speed) and the sharpness of the contours. To overcome these weaknesses, a condition is supposed that in the energy functional different regions have different contribution. We alleviate it by using the image entropy to add coefficients to two elements of the traditional energy functional together with normalization for the integrals in Eq. 6. The main reason is that cuts usually favor equal size regions. For images where the foreground is larger or smaller than the background, it is more efficient to use normalized cuts. Thus, the local energy term is rewritten as following

$$E_{local}(C) = \frac{E_{in}}{A_{in}} \iint_{R_i(C(t))} \iint_{R_o(C(t))} \omega(u, v) du dv + \frac{E_{out}}{A_{out}} \iint_{R_o(C(t))} \iint_{R_i(C(t))} \omega(u, v) du dv \quad (9)$$

Minimizing the proposed energy function in Eq. 5, we can find a contour C such that the below goals could be achieved:

- The image is partitioned into two regions (inside and outside the curve C) such that the dissimilarity within each region are minimized; and
- The Bhattacharyya distance between these two regions is maximized.

To deal with topological changes, we transform this energy functional into the level set formulation. Lastly, minimization of the energy function ϕ in Eq. 5 is to derive the below evolution flow equation

$$\frac{\partial \phi}{\partial t} = \delta_x(\phi) \left\{ -\beta \left(\frac{E_{out}}{A_{out} R_i} \iint \omega(u,v) dudv + \frac{E_{in}}{A_{in} R_i} \iint \omega(u,v) dudv \right) - (1-\beta) \sum_{i=1}^3 \left[\frac{B^i}{2} \left(\frac{1}{A_{in}} - \frac{1}{A_{out}} \right) + \frac{1}{2} \int_z \delta(z-l^i) \left(\frac{1}{A_{out}} \sqrt{\frac{P_{in}^i}{P_{out}^i}} - \frac{1}{A_{in}} \sqrt{\frac{P_{out}^i}{P_{in}^i}} \right) dz \right] \right\} \quad (10)$$

where A_{in} and A_{out} are given by

$$\begin{aligned} A_{in} &= \int_{\mathbb{W}} H(-f(x)) dx \\ A_{out} &= \int_{\mathbb{W}} H(f(x)) dx \end{aligned} \quad (11)$$

Note that in digital images, the two probability distributions P_{in}^i and P_{out}^i in Eq. 10 are simply the histograms inside and outside the curve in the i^{th} color channel.

At each the iteration deformation, ϕ is updated its values. Not considering ϕ in a whole image, our algorithm only focus on pixels in which its value changes the sign. Thus, only sign-changed pixels within the contour neighborhood are replaced by its new value until the object of interest is achieved completely. Other surrounding objects which are far away but have color similarity are easily eliminated if the contour is split and attracted to their locations by the level set framework.

4. EXPERIMENTAL RESULTS

For all experiments in this work, the proposed method was implemented in MATLAB version 7.12.0 (R2011a). Some functions related to GPAC code written by Luca Bertelli [21] in C++ programming language and compiled as the *mex* files are also used. This program runs on a PC equipped with an Intel® Core™2 CPU 6700 at 2.66 GHz, 2.67 GHz and 2 GB of RAM. All fix parameter values in Eq. 10 is in turn set as $\varepsilon = 0.001$, $\nu = 4000$, $\beta = 0.5$, $\mu = 0.04$, and the time step $\Delta t = 0.5$. The convergence is achieved if the number of sign-changed energy value of pixels is smaller than a threshold of 20 pixels.

We evaluated the performance of the proposed algorithm on 104 natural color images collected from Achanta et al. [24], Berkeley Segmentation Dataset (BSDS500) [25], and Microsoft Research Cambridge (MSRC) [26, 27] for our specific problem. Because of the complexity of the algorithm, all images are reduces their size to be smaller than 400 pixels in each dimension (height and width).

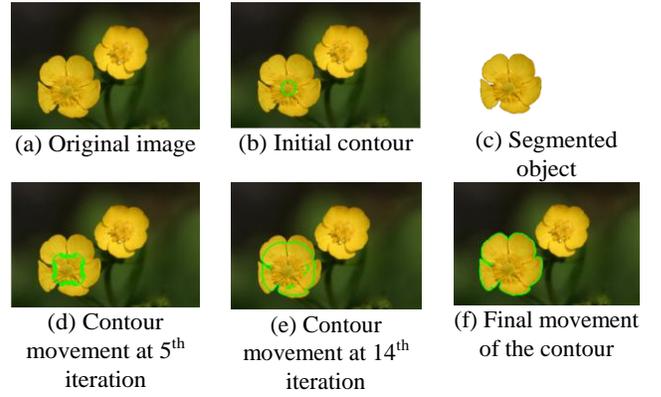


Fig. 4. Object of interest segmentation process by our location-based level set method

Fig. 4 visualizes the evolution of our proposed segmentation algorithm applied on a 400×266 color image. The original image is **Fig. 4(a)** and the initial zero level set is drawn as a green circle in **Fig. 4 (b)**. Thus we look forward to segment the biggest yellow flower. The final result is depicted accurately in **Fig. 4 (c)**. **Fig. 4 (d)**, **(e)**, and **(f)** are, in turn, results obtained at 5th, 14th, and final step of the propagation, respectively. It totally takes around 17 seconds. The final contour looks quite smooth and almost fit to the concerned object. Especially, the algorithm can separate our considered flower from the neighboring one due to the distance constraint added to the dissimilarity measure in Eq. 8.

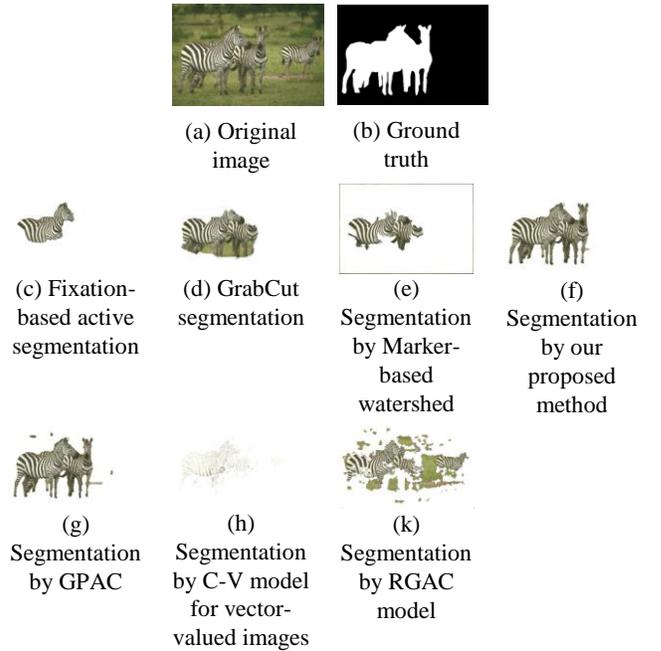


Fig. 5. Visual comparison of various segmentation methods

Next figure illustrates a visual comparison between the output of our proposed method and six different segmentation approaches. These algorithms include the marker-based watershed [28], GrabCut [29], fixation-based active segmentation [30], Graph partitioning active contour (GPAC) [20], RGAC model [31], and Chan-Vese model for vector-valued images [32]. In this figure, the segmentation results are given in **Fig. 5 (c-k)** respectively. As shown in **Fig. 5**, our location-based ACM gives the most exact segmentation rather than other methods in comparison with its

ground truth image.

Table 1 Comparison of user interactive segmentation methods

Method	Precision	Recall	F-measure
Marker-based watershed	0.66	0.84	0.73
GrabCut	0.79	0.84	0.80
Fixation-based active segmentation	0.94	0.72	0.75
GPAC	0.79	0.78	0.75
RGAC	0.50	0.77	0.60
C-V model for vector-valued image	0.58	0.58	0.59
Our proposed method	0.86	0.82	0.81

Table 1 summarizes the performance of various segmentation methods applied to whole our image database through quantitative metrics (i.e. average precision, recall and F-Measure). The results show that the proposed method can successfully segment only one object of interest in many natural color images with a lower false-positive rate or a higher precision. With other level set-based ACMs in the comparison, because they do not consider the location relationship between pixels and the initial region, they are very sensitive to colors and therefore produce many false positives. Compared to various user interactive algorithms, our proposed method generally achieves good performance as well.

5. CONCLUSIONS

In this paper, a level set method based on location information to deal with color interested objects segmentation has been presented. The integration of global and local energy terms in the energy functional is proposed to propagate the initial contour towards boundaries of the object as well as save its convergence after a finite number of iterations. The experiments show that our suggested method generates promising results on various natural images. In the future we will optimize its processing time and improve its energy functional for a more accurate segmentation in complex cases.

6. ACKNOWLEDGMENTS

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